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Abstract. Some ways in which gravitational lenses can act as crude telescopes are reviewed. Magnification limits associated with finite source size, corrugation of the gravitational potential and finite wavelength are specified. It should be possible to obtain rotation curves for a small sample of ultra-faint galaxies imaged as giant arcs. There is an appreciable chance of eventually observing a radio jet component cross a caustic. The successful observation of microlensing in 2237+031 suggests that the continuum emission from this source at blue wavelengths is at least partly non-thermal. The maximum possible magnification observable, granted existing constraints is argued to be from γ-ray pulsars crossing caustic sheets formed by galaxies or a hypothetical population of intergalactic massive objects.

1. Magnification near caustics

1.1. FOLDS

It is convenient, though not obligatory, to illustrate large magnification effects using a weakly elliptical gravitational lens (*eg* Blandford & Kovner 1988, Schneider et al. 1992). Suppose that the scaled potential can be written in the form

$$\Phi(\vec{r}) = f(r) + \psi(r, \phi) \tag{1}$$

where we fix the angular radius of the unperturbed Einstein ring to be $r = b$ by setting $f'(b) = b$ and treat ψ as a perturbation. To first order, the source is located at

$$\vec{\beta} = b\, h_r\, \delta\, \vec{\hat{r}} - \nabla\psi \tag{2}$$

where $\delta = r - b$ is the radial displacement of the image from the unperturbed Einstein ring, $h_r = 1 - f''(b)$ and $\nabla\psi = \nabla\psi(b, \phi)$, etc. A small circular

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source of angular radius a will be magnified into an elliptical image with minor axis of length a/h_r and major axis a/h_t where $h_t = b^{-1}\delta h_r - b^{-1}\psi_{,r} - b^{-2}\psi_{,\phi\phi}$, rotated with respect to the radial direction by an angle $(b^{-2}\psi_{,\phi} - b^{-1}\psi_{,r\phi})/h_r$. The magnification becomes infinite on the critical curve where $h_t = 0$. This is displaced from the unperturbed Einstein ring by $\delta_c = (\psi_{,r} + b^{-1}\psi_{,\phi\phi})/h_r$. (The corresponding displacements of the equipotential and isodensity contour are $\delta_\Phi = -\psi/b$ and $\delta_\Sigma = (\psi_{,r}+b\psi_{,rr}+b^{-1}\psi_{,\phi\phi})/(h_r - bf''')$.) This critical curve maps onto the caustic in the source plane.

For the simple case of an isothermal potential with $f(r) = br$ and an external tidal perturbation, $\psi = \epsilon r^2 \cos(2\phi)$, the contours of constant surface density are circular. (This can occur if the tidal force cancels the centrifugal force, for example.) The radial magnification is unity and the critical curve has an ellipticity 4ϵ (in the epicyclic approximation). The tangential caustic is an astroid with four cusps located at $(\pm 4\epsilon b, 0), (0, \pm 4\epsilon b)$. The total cross section for magnification of a stationary source at a fold by more than $\mu \equiv 2/h_t$ (including both images) can be computed to be $2/\mu^2$ times the area of the Einstein ring (for $\mu >> 1$), and if the source approaches a fold with a proper motion $\dot{\theta}$, then the observed magnification will vary with time according to $\mu = (\pm 3\epsilon\dot{\theta}|t|/b)^{-1/2}$. The total caustic length on one side of the caustic is $24\epsilon b$.

1.2. CUSPS

Three bright images can be observed when the source lies inside a cusp. In addition, as the contours h_t = constant in the source plane encircle the cusp point and cross themselves, there is a small zone of dimensions $\sim h_t \times h_t^{3/2}$ lying just outside the cusp within which there will be one image formed with magnification $\mu > (h_r h_t)^{-1}$. The integral cross section for cusp magnification diminishes as $h_t^{5/2} \propto \mu^{-5/2}$ and so cusps are less likely to be involved in the highest magnification events of stationary sources than folds. At high enough magnification, the caustic can be treated locally like a fold except in the single image region, where the length scale over which a cusp magnification in excess of μ is created is $\sim 4/\mu$. Hence at high enough magnification ($\mu \gg \epsilon$), we expect that folds will dominate. Similar remarks apply *a fortiori* to the next simplest catastrophe, the hyperbolic umbilic, though here it is more useful to think of it as a singular point at the intersection of a pair of intersecting caustic sheets.

1.3. MAGNIFICATION LIMITS

A point source can be magnified by an arbitrarily large factor close to a caustic. In practice, three distinct limitations are relevant.

1.3.1. *Source size limit*

The fold magnification equation will break down when the source radius a is comparable with its distance from the caustic, *ie* when

$$\mu \sim \mu_{\rm SS} \equiv \left(\frac{b}{\epsilon a}\right)^{1/2} \tag{3}$$

(where b is now introduced to be the Einstein radius). Note that the larger and more circular the lens, the greater the possible magnification. For a cusp, the peak magnification scales $\propto a^{-2/3}$ and so a greater magnification is possible for a given source.

1.3.2. *Corrugation limit*

This limiting magnification at a fold is inversely proportional to the third derivative of the potential along the direction of maximal linear magnification. If there are additional perturbations, for example those associated with individual galaxies in the case of clusters and individual stars in the case of galaxy lenses, the maximum magnification may be reduced. Suppose that a fraction $f(M)$ of the mass contained within the Einstein ring M_E, is contained in uncorrelated substructures of mass M. The closest such structure is expected to be at a distance $\sim (M/fM_E)^{1/2}b$ and it will dominate the third derivative at the new location of the critical curve and create $\sim f/f_{\rm crit}$ extra images and increase the caustic length if $f \gtrsim f_{\rm crit} \sim (\epsilon^2 M/M_E)^{1/3}$. In this case, fluctuations out to a distance $\sim (fM/\epsilon^2 M_E)^{1/4}b$ will also contribute and the potential will only be smooth on larger scales. This limits the magnification to

$$\mu \sim \mu_{\rm CL} \equiv \mu_{\rm SS}\left(\frac{f}{f_{\rm crit}}\right)^{-3/4}; \qquad a \lesssim \frac{f^{1/2} f_{\rm crit}^{3/2} b}{\epsilon} \tag{4}$$

If there is substructure on more than one mass scale then this limitation to the magnitude must be applied hierarchically.

1.3.3. *Geometrical optics limit*

Geometrical optics can only be used within the diffraction limit, which depends upon the Fresnel length (*eg* Gould 1992, Ulmer & Goodman 1995). This limits the magnification to

$$\mu \sim \mu_{\rm GO} \equiv \left(\frac{D}{\lambda}\right)^{1/3}\left(\frac{b}{\epsilon}\right)^{2/3} \tag{5}$$

2. Faint Galaxies

2.1. BASIC PROPERTIES

A remarkable discovery of recent years is the large excess of faint galaxies (Tyson 1988). So far, faint galaxy counts on the sky have reached 30 billion (Smail et al. 1995) and the number appears to be still rising at a rate of 2 per magnitude. Another way to express this is to note that the faint galaxies are, on average, about $4''$ (or ~ 5 kpc) apart, much smaller than a nominal galaxy size. It is found that the half power radii of these faint galaxies diminishes with decreasing flux to become $\sim 0.2''$ for the faintest galaxies.

2.2. CLUSTER ARCS

Cluster arcs provide a natural way to image the faintest galaxies. The best example to date is A 2218 (Kneib et al. 1995) and here there are over ten well magnified structures that have been resolved in both directions by HST. Typically these arcs are observed over several thousand pixels. It should then be possible to use these observations to solve for the source structure. So far this program has only be carried out using model-fitting (*eg* Worthey et al.'s (1995, preprint) analysis of 0024+1654). However the HST data are sufficiently fine scale that it is worth developing techniques analogous to those used at radio frequencies to invert the images and obtain accurate source structure so that significant morphological information can be extracted.

The next step will be to perform one-dimensional imaging spectroscopy on these arcs (*cf* Pello et al. 1991) using large aperture, ground-based telescopes. In this way it might be possible to extract dynamical information and, given sufficient examples, determine the evolution of empirical relations like the Faber-Jackson relation. The limiting magnification is likely to be corrugation-limited by the individual cluster galaxies which comprise a fraction $f(10^{11}M_\odot) \sim 0.1$ of the total cluster mass within the Einstein ring. In this case, $f_{crit} \sim 0.02$ and we expect that the magnification will be limited to $\mu_{max} \sim 10 - 30$ as, indeed, is observed.

Cluster arcs can exhibit other high magnification effects such as triple imaging of supernovae (Kovner & Paczyński 1988). If the cluster potential were smooth, it would be possible to use the relative magnifications and time-delays to check fundamental scaling relations for cusps, (*eg* the central image flux should equal the sum of the fluxes from the outer images etc). However, it is more likely to provide information about the potential corrugation near the critical curve. Of most importance, though, for our topic is that as there can be notice of a supernova at high redshift which can be monitored intensively during its second or third appearance. In principle, such observations can provide a measurement of Ω_0 using a supernova that would ordinarily be too faint to observe if it were unmagnified.

3. Radio Jets

3.1. BASIC PROPERTIES

Active Galactic Nuclei (AGN) can be conveniently divided into the radio-quiet and radio-loud. The latter class are traditionally sub-divided into compact and extended sources, depending upon whether or not the dominant emission at an intermediate radio frequency comes from the nucleus of the galaxy (or quasar) or radio lobes external to the galaxy (*eg* Zensus & Pearson 1990). It appears that essentially all powerful radio sources are fueled by a pair of antiparallel, relativistic jets that beam their radio synchrotron emission along their directions of motion. According to the radio unification hypothesis, compact and extended sources belong to the same family with the compact sources being those that are beamed towards us. An extension of this hypothesis posits that the powerful radio galaxies are radio-loud quasars observed at latitudes $\lesssim 45°$ so that their broad emission line regions are obscured (*eg* Antonucci 1993). Radio jets in compact sources often exhibit core-jet structure and "superluminal expansion". The cores are commonly interpreted as the self-absorbed inner regions of the jets and, consequently have flat radio spectra. The jet emission is optically thin and is probably associated with outwardly propagating relativistic shock fronts.

In recent years, it has been discovered that many compact radio sources exhibit "intraday variability" at cm wavelengths. The significance of this observation is that if one takes the variability time scale as a measure of the light crossing time across the emitting region, then the derived radio brightness temperature can exceed $\sim 10^{18}$ K. This is six orders of magnitude larger than the so-called "inverse Compton" limit. As is well known this limit can be exceeded if there is relativistic expansion. However, the bulk Lorentz factors required are typically ~ 100 and the jet flows are radiatively inefficient. Alternatively, the variability may be due to refractive scintillation (*eg* Rickett et al. 1995). Several authors have proposed that the emission mechanism is coherent and, consequently, that there is no inverse Compton limit. Here, however, there is a new difficulty in that it is very difficult for radio emission with brightness temperatures as high as $\sim 10^{18}$ K to emerge from an AGN. A very small Thomson optical depth suffices to degrade the radio beam through a combination of induced Compton scattering and stimulated Raman scattering (*eg* Levinson & Blandford 1995).

3.2. MAGNIFICATION OF RADIO JETS BY INTERVENING GALAXIES

Using galaxy lenses as radio telescopes may clarify some of these issues. Large surveys like CLASS and WENSS should discover tens of multiply-imaged compact sources and the cores of the best cases will undoubtedly be monitored with the VLBA (Myers, these proceedings). Typical magnifications for cores are $\mu \sim 10$, so we ought to resolve them under the synchrotron hypothesis. If there is superluminal expansion with $V_{obs} \sim 10c$, then this will be seen as "hyperluminal expansion" with $V_{obs} \sim 100c$. However, we will not be able to measure higher brightness temperatures. For the brightest observed compact sources, with flux densities $S \sim 10$ Jy, these are limited by $T_{Bmax} \sim SR_{\oplus}^2/k \sim 10^{12}$ K (Levy et al. 1989). As observed core sizes appear to be wavelength-dependent, they will be subject to differential magnification (Connor et al. 1992, Nair, these proceedings).

It would be of great interest to use a galaxy lens to super-resolve a compact radio core. The most information would come from mm VLBI observations. For a source size $a \sim 10^{17}$ cm and $\epsilon \sim 0.1, b \sim 3$ kpc, the maximum observable linear magnification should be $\sim 10^3$. However, in order for a gravitational telescope to achieve this magnification requires that we find a source close enough to a fold caustic. If we adopt a transverse velocity ~ 1000 km s^{-1}, a mm core will only move its own diameter in ~ 30 yr. Therefore adopting the magnification cross section and allowing for selection effects, the maximum magnification of a mm core that we are likely to find is no more than several times the square root of the number of radio rings we discover, at most 30 unless we are very lucky. It is however possible that some of the observed intraday variability is attributable to microlensing variation associated with intervening stars. If there really are ultra-compact coherent sources within radio cores then the $\sim 10\mu$s time delays associated with stellar microlensing may be detectable by autocorrelating the radio signal (Moore & Hewitt, these proceedings).

The probabilities are somewhat greater for moving features in a jet as these extend to much greater distances particularly at lower radio frequency, typically ~ 10 pc. As the characteristic linear size of a tangential caustic is ~ 300pc, there is a reasonable chance that in one of the radio rings to be discovered, a jet will straddle a fold caustic and features will be seen crossing it on a regular basis.

4. Accretion Disks

4.1. BASIC PROPERTIES

Radio-quiet quasars, and their lower luminosity cousins, the Seyfert galaxies, represent the majority of AGN that do not appear to possess prominent relativistic jets. The popular supposition, for which there is some observational support, is that these are equatorial outflows of gas with speeds up to $\sim 30,000$ km s^{-1}, at least in the case of the quasars. When the observer is located in the equatorial plane a broad absorption line quasar (BALQ) is seen; otherwise the object is classified as a radio-quiet quasar (*eg* Weymann et al. 1991).

AGN in general are commonly interpreted in terms of the black hole - accretion disk model. The circumstantial evidence in favor of this interpretation has accumulated and includes the large required efficiency, observed rapid variability and dynamical estimates of the central mass, most recently from observations of H_2O masers in M100 (Miyoshi et al. 1995), from X-ray line profiles in MCG-6-30-15 (Tanaka et al.) and also from M87 (Harms et al. 1994). Typical black holes masses are believed to lie in the $10^6 - 10^9$ $M_\odot$ range, so their Schwarzschild radii are $\sim 3 \times 10^{11} - 3 \times 10^{14}$ cm. We now have better grounds to believe that the photoionizing UV and X-ray continuum originates from a very compact region close to the central black hole, presumably the surface of an accretion disk. However, the failure to detect predicted signatures of accretion disks such absorption edges, large linear polarization and radial color variation is troubling (*eg* Antonucci 1993).

The broad emission line clouds, when present, are now thought to be located over a broad range of radii dependent upon the ionization state, but centered roughly on $\sim 0.1 L_{46}^{1/2}$ pc. This can be estimated using the technique of reverberation mapping (Peterson *et al* 1995). In the standard model, a typical cloud has size $\sim 10^{13}$ cm and density $\sim 10^{10}$ cm^{-3}. The filling factor of the emitting phase is generally very small $\sim 10^{-5}$. The dynamical state and provenance of these clouds (moving with internal Mach numbers $\gtrsim 1000$ is problematic, though there are several models (*eg* Netzer 1991).

The broad absorption line gas in quasars is believed to be located exterior to the broad emission line region at typical radii $\sim 10^{19}$ cm. Photoionization arguments suggest densities $\sim 10^8$ cm^{-3} and individual cloud sizes of only $\sim 10^9$ cm (*eg* Weymann et al. 1991).

4.2. MICROLENSING OBSERVATIONS

One of the most interesting uses of gravitational telescopes has been indirectly in observing quasars that appear to be microlensing, most notably 2237+031. Here, the important deduction is that, in order to exhibit the observed variable magnification, the size of the accretion disk, believed to be responsible for the continuum emission must satisfy the source size limit for an individual star ($\sim 1 - 3 \times 10^{15}$ cm; neither the emission line region nor the compact radio sources are likely to be be microlensed and this is one test that has been applied to validate claims of optical microlensing.) In the case of 2237+031, this must happen at the longest wavelength for which microlensing variation has been reported (*ie* 1μ or ~ 4000 Å in the quasar frame), and the black hole must be massive enough and the accretion disk consequently large enough to account for the total bolometric luminosity of the object, corrected for macrolensing by the intervening galaxy. This, in turn, suggests that some of the optical emission may be non-thermal. (See Rauch 1993 for a review of this topic.) In the future, it may be possible perform "microlens mapping" to measure the size of the emitting region as a function of wavelength (Wambsganss & Paczyński 1994). However, the simplest interpretation of the results of reverberation mapping of Seyferts suggests that the size of the emitting region is wavelength-independent (Peterson et al. 1995).

In a variant on this approach, Francis & Koratkar, (these proceedings), have reported that high redshift quasars have lower equivalent widths than low redshift quasars, an effect that they attribute to stellar microlensing preferentially magnifying the continuum. If this interpretation turns out to be correct, it introduces a much larger sample of microlensed sources, whose internal structure can be studied statistically.

4.3. PROBING THE BROAD ABSORPTION LINE REGION

Another interesting possibility is suggested by the observation that at the putative distance of the broad absorption line gas, the continuum source subtends an angular size $\sim 1''$. Three gravitationally lensed quasars are reported to exhibit broad absorption lines UM425, 1115+080 (Michalitsianos & Oliverson, these proceedings) and HR 1413+117 (Kayser et al. 1990) and the typical rays are separated in angle by a comparable amount. Therefore, it is possible that by studying the absorption troughs associated with the individual images we can probe length scales $\sim 10^{14}$ cm in the absorbing region.

4.4. BLACK HOLE LENSES

The ultimate gravitational lens is undoubtedly a black hole and the caustics associated with the Kerr spacetime and a different observer have been calculated by Rauch & Blandford (1994). The best prospects for seeing high magnification are if there is a population of X-ray-emitting, accreting neutron stars orbiting the black hole at high speeds $\sim 0.03c$. Source size-limited magnifications of several thousand are possible, but the caustic crossing rate will be low and difficult to distinguish from regular X-ray variability.

5. Pushing the Envelope

Finally, I address the question of what is the largest magnification that we could imagine observing, granted the physical constraints listed above and observational limitations on sources and lenses (*cf* Miralde Escudé 1991). We are most interested in highly compact sources that satisfy the geometrical optics constraint. This points us towards γ-ray energies. The most compact, steady γ-ray sources that we know of are γ-ray pulsars, where the size of the emitting region is argued to be $a \sim 10^6$ cm. For the lenses, we can choose between clusters (with $M_E \sim 10^{14}$ $\mathrm{M}_\odot$, $b \sim 100$ kpc, $\epsilon \sim 0.1$) and galaxies with ($M_E \sim 10^{11}$ $\mathrm{M}_\odot$, $b \sim 3$ kpc, $\epsilon \sim 0.1$). In the former case we suppose that there is an intergalactic population of stars with $f \sim 0.01$ so that $f_{\rm crit} \sim 10^{-5}$. Typically the peak magnification is limited to $\mu_{\rm CL} \sim 3 \times 10^6$, while the total length of fold caustic has been lengthened roughly a thousand times to $\sim$ 100 Mpc per cluster. In the latter case, we are in the strong microlensing limit with $f \sim 1$ and the stars are not perturbative. In the absence of a relevant simulation we retain the perturbative approach and estimate that $f_{\rm crit} \sim 10^{-4}$ and that the caustic length is $\sim$ 30 Mpc per galaxy but that $\mu_{\rm CL} \sim 3 \times 10^5$. (A hypothetical population of intergalactic $\sim 10^{10}$ $\mathrm{M}_\odot$ black holes would produce comparable effects with less caustic length but higher magnification.) Furthermore, the geometrical optics constraint requires that $\lambda \lesssim 1$ Å for maximal magnification and so the observed flux will be diminished at and below X-ray energies.

All of this is reminiscent of the observed properties of γ-ray bursts which occur at a rate of $R_b \sim 10^{-5}$ s^{-1} with a fluence $E_b \sim 10^{-7}$ erg cm^{-2}. It is possible that some observed bursts are actually persistent γ-ray sources crossing galaxy or cluster caustic surfaces. There are however, two important limitations. Firstly, the total γ-ray flux from the totality of unmagnified sources should not exceed the general γ-ray background flux, $F_{grb} \sim 4 \times 10^{-7}$ erg cm^{-2} s^{-1}. Secondly the intrinsic source luminosities must be large enough that we can see them at the observed fluences. In other words, if the burst duration is t, then $E_b \sim (L_\gamma/4\pi D^2)(b/\epsilon a)^{1/2}(f/f_{\rm crit})^{-3/4}t$. If the optical depth to multiple imaging is τ, and the number of bright γ-ray sources is N_γ, then the expect burst frequency satisfies

$$R_b \sim \frac{N_\gamma \tau}{\pi b^2}\frac{24\epsilon a b}{t}\frac{f}{f_{\rm crit}} \lesssim 24\frac{F_{grb}}{E_b}\left(\frac{\epsilon a}{b}\right)^{1/2}\left(\frac{f}{f_{\rm crit}}\right)^{1/4}\tau \tag{6}$$

On this rough basis, the galaxy-induced frequency is $R_b \lesssim 10^{-8}$ s^{-1} and the cluster-induced rate is somewhat smaller. Intergalactic black holes could lead to a larger burst frequency $R_b \lesssim 10^{-7}$ s^{-1}. No more than a minority of observed bursts are likely to be formed in this manner.

There is an interesting way to recognize caustic induced bursts. Interplanetary spacecraft locate bright bursts by timing a plane wave crossing three detectors presumably at the speed of light. Two solutions can be found and one of these is usually found to agree with other directional information. However if a caustic sheet passes through the spacecraft, the deduced wave normal should be orthogonal to the source location. (In a simple lens this caustic will move much slower than the speed of light. However, relative motions in the lens can lead to much faster caustic motion, (*eg* Wambsganss, these proceedings). A search for examples of caustic-induced bursts would be of interest.

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